

Chapter 28. The Chi-Square Test

1. (a) (i) (b) (iii). See exercises 3–6 on pp.539–40.
2. Use the method of sections 1–2.

Observed	Expected
1	17.6
10	30.1
16	7.4
35	6.9

$\chi^2 \approx 150$  on 3 degrees of freedom, so  $P \approx 0$  and option (ii) is right.

Comments. (i) Judges prefer well-educated grand jurors.

(ii) The expecteds do not have to be whole numbers. For instance, if you roll a die 100 times, the expected number of aces is 16.666 . . . .

3. Use the method of section 4. The expecteds are as follows (row and column sums are a bit off due to rounding).

	Married	Widowed, divorced, or separated	Never married
Employed	772.0	103.3	221.7
Unemployed	66.2	8.9	19.0
Not in labor force	28.9	3.9	8.3

$\chi^2 \approx 14.2$  on 4 degrees of freedom, which is off the end of the table. By computer,  $P \approx 0.7$  of 1%. This is not chance variation. The married men do better at getting jobs. (Or, men with jobs do better at getting married: the  $\chi^2$ -test will not tell you which is the cause and which is the effect.)

4. (a) Chance: it's a probability histogram.
- (b) The chance that  $5 \leq \chi^2 < 5.2$ , where  $\chi^2$  is computed from 60 rolls of a fair die.
- (c) The chance that  $5 \leq \chi^2 < 5.2$  is bigger than the chance that  $4.8 \leq \chi^2 < 5$ . (The block is bigger.)
- (d) 10%.

Comment. The exact probability distribution of  $\chi^2$ , with 60 rolls, is quite irregular. As the number of rolls goes up, the histogram gets closer to the curve. See p.41 of this manual.

5. (a) With 10 degrees of freedom,  $P$  will be bigger. Reason: that curve has more area to the right of 15.
- (b) The  $P$ -value is bigger when  $\chi^2 = 15$ . Reason: the area under the curve to the right of 15 is bigger than the area to the right of 20.
6. Use the method of section 3. The observed frequencies are too close to the expected ones for comfort:  $\chi^2 \approx 2$  on 10 degrees of freedom, so  $P < 1\%$  (left tail); by computer,  $P \approx 0.4$  of 1%. This individual seems to have very good control over the dice. Maybe you should decline his invitation to play craps.
7. Use the method of sections 1–2:  $\chi^2 \approx 0.2$  on 2 degrees of freedom,  $P \approx 90\%$ , a good fit.

8. Make a  $\chi^2$ -test, as in sections 1–2. We are interested in the chances, not just the average:  $\chi^2 \approx 2.6$  on 5 degrees of freedom,  $P \approx 75\%$ , a good fit.
9. Use the method of section 4. The expecteds are as follows:

	20–24	25–29
Never married	34.9	32.1
Married	25.5	23.5
W/D/S	3.6	3.4

$\chi^2 \approx 17$  on 2 degrees of freedom; by computer,  $P < 2/1000$ . This is not chance variation. It takes time to get married, especially in Montana.

10. (a) Use the method of section 4 to compute  $\chi^2$ .

	Observed		Expected	
	Protestant	Catholic	Protestant	Catholic
Acquitted	8	27	6.56	28.44
Convicted	7	38	8.44	36.56

	Obs – Exp	
	Protestant	Catholic
Acquitted	1.44	-1.44
Convicted	-1.44	1.44

Now

$$\begin{aligned}\chi^2 &= \frac{1.44^2}{6.56} + \frac{1.44^2}{8.44} + \frac{1.44^2}{28.44} + \frac{1.44^2}{36.56} \\ &= 1.44^2 \times \left( \frac{1}{6.56} + \frac{1}{8.44} + \frac{1}{28.44} + \frac{1}{36.56} \right) = 0.69\end{aligned}$$

and  $P \approx 60\%$ . The mistake, apparently, was to compute

$$1.44^2 \div \left( \frac{1}{6.56} + \frac{1}{8.44} + \frac{1}{28.44} + \frac{1}{36.56} \right) = 6.22.$$

- (b) Presumably, the defense was thinking that accused persons are convicted independently, with a common probability—except that there is one probability for Catholics and another for Protestants. This model does not seem well related to the criminal justice system, where the facts vary from one case to the next, and some cases involve multiple defendants.